PB86-171659

Diffusion in the Stably Stratified Atmospheric Boundary Layer

American Meteorological Society, Boston, MA

Prepared for

Environmental Protection Agency
Research Triangle Park, NC

1985

U.S. Department of Commerce
National Technical Information Service
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TECHNICAL REPORT DATA
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1. REPORT NO. EPA/600/J-85/329	2.
3. RE PB86-171659	
4. TITLE AND SUBTITLE: DIFFUSION IN THE STABLY STRATIFIED ATMOSPHERIC BOUNDARY LAYER (Journal Version)	5. REPORT DATE 1985
	6. PERFORMING ORGANIZATION CODE
7. AUTHOR(S): J. C. R. Hunt	8. PERFORMING ORGANIZATION REPORT NO.
9. PERFORMING ORGANIZATION NAME AND ADDRESS: American Meteorological Society, Boston, Massachusetts	10. PROGRAM ELEMENT NO. CDWA1A/02 0279 (FY86)
	11. CONTRACT/GRANT NO. CR-810297-02
12. SPONSORING AGENCY NAME AND ADDRESS: Atmospheric Sciences Research Laboratory - RTP, NC, Office of Research and Development, U.S. Environmental Protection Agency, Research Triangle Park, NC 27711	13. TYPE OF REPORT AND PERIOD COVERED
	14. SPONSORING AGENCY CODE EPA/600/09

15. SUPPLEMENTARY NOTES

J. Climate and Appl. Meteorol., 24:1187-1195, Nov. 1985

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The effects of a shear on vertical and lateral diffusion are briefly discussed and also the lateral spreading of initially well-mixed plumes.

17. KEY WORDS AND DOCUMENT ANALYSIS

a. DESCRIPTORS	b. IDENTIFIERS/OPEN ENDED TERMS	c. COSATI Field/Group

REPRODUCED BY
NATIONAL TECHNICAL
INFORMATION SERVICE
U.S. DEPARTMENT OF COMMERCE
SPRINGFIELD, VA. 22161

18. DISTRIBUTION STATEMENT: RELEASE TO PUBLIC	19. SECURITY CLASS *(This Report)* UNCLASSIFIED	21. NO. OF PAGES 11
	20. SECURITY CLASS *(This page)* UNCLASSIFIED	22. PRICE

EPA Form 2220-1 (Rev. 4-77) PREVIOUS EDITION IS OBSOLETE

EPA/600/J-85/329
JOURNAL ARTICLE

Diffusion in the Stably Stratified Atmospheric Boundary Layer

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(Manuscript received 2 July 1984, in final form 30 April 1985)

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The assumptions, techniques and methods of improving Gaussian plume modeling are briefly reviewed. Then the statistical theory for the diffusion of a substance S into a turbulent flow in homogeneous and inhomogeneous flows, and neutral and stably stratified flows is introduced. In stably stratified flows the mixing of the temperature and the substance S *between* fluid elements is of critical importance in determining the growth of a plume, so that the plume's depth is *not* in general determined by the displacements of fluid elements through the source, nor by the conventional Lagrangian statistics. But the plume depth, far downwind of the source, can be related to the local thermal diffusivity in the atmospheric boundary layer. The conditions are deduced under which this result can be used in stable conditions. The structure of turbulence in the stable boundary layer (SBL) is discussed. It is concluded that it is probably too variable to be characterized reliably by typical profiles of the turbulence and mean velocity for general air-pollution dispersion calculations, in the same way as the convective boundary layer can be characterized. It is also shown that the observed variability of the SBL is consistent with the observed variations in the growth rates of the plumes.

The effects of a shear on vertical and lateral diffusion are briefly discussed and also the lateral spreading of initially well-mixed plumes.

1. Introduction

Recently, new facilities for laboratory and atmospheric experiments have provided new measurements on turbulence and diffusion in stably stratified turbulent flows and the stable boundary layer (SBL) (Dickey and Mellor, 1980; Britter *et al.*, 1983; Stillinger *et al.*, 1983; Caughey *et al.*, 1978; Nieuwstadt 1982; Hunt *et al.*, 1985). There are now on-line instruments for continuous monitoring of many aspects of atmospheric motion and mixing. These developments are stimulating new theory and computations which should enable the modeling of atmospheric diffusion to be improved. Some suggestions have already been made by Weil (1983) and Hunt (1982).

The basis of the current practice of "Gaussian plume modeling" is to assume

i) the meteorological conditions that determine turbulent diffusion from ground-level sources can be characterized by some index or parameter (e.g., the Pasquill letter categorizes A to G), usually calculated from measurements made only at ground level;

ii) for each such condition the vertical and horizontal distribution can be assumed to have the form of "Gaussian" profiles with widths σ_z, σ_y which are specified as a function of distance (largely on the basis of experiments); σ_y is assumed to be independent of height;

iii) the same index or parameter is appropriate for defining meteorological conditions and thence the diffusion from elevated sources;

iv) the growth rate of plumes from elevated sources is approximately the same as ground-level sources, provided an image source is placed below the ground. (Other images may be used for elevated inversions.)

There are a variety of different ways of improving the predictions of turbulent diffusion. At one extreme one might use a limited number of atmospheric measurements (e.g., at the surface only), and rely on extensive computations (or correlations of empirical data or computations of approximate equations) to infer other properties of atmospheric turbulence and diffusion (Fig. 1). At the other extreme, extensive on-line monitoring (using remote sensors) of atmospheric motions, including the turbulence, at all the appropriate levels might be used to infer turbulent diffusion directly, with the minimum of computation of the boundary-layer dynamics. It is likely that different approaches may be necessarily depending on how complex the flows are at the relevant locality (in the SBL this does not mean the terrain has to be complex), and depending on the availability of on-line monitoring. In either case, it is important to estimate the likely *range* of plume widths to be expected for any given atmospheric condition. (This range is a measure of our lack of both understanding and uncertainty in defining the flow.)

In section 2 we outline some principles of turbulent diffusion in stably stratified turbulent flows, showing how diffusion is related to the local properties of the

J. Climate and Appl. Meteorol., 24: 1187-1195

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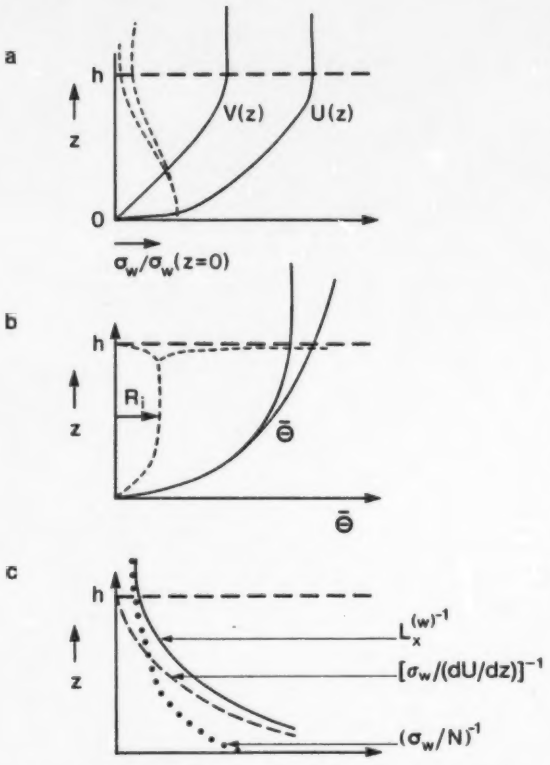

FIG. 1. (a) Typical profiles in the stable boundary layer; mean velocity components parallel U and perpendicular V to the surface wind. The rms vertical turbulence σ_w, when $\sigma_w \to 0$ at $z \geq h$ and when $\sigma_w \neq 0$ indicated by dashed line. (b) Typical profiles of mean potential temperature and gradient Richardson number Ri, depending on whether the air above the SBL is neutral (typical in early evening) or stable (typical in the middle of the night). (c) Profiles of the reciprocal of the integral scale of the vertical component of turbulence $L_x^{(w)-1}$ in the SBL compared with the reciprocals of the buoyancy scale $(\sigma_w/N)^{-1}$, and velocity gradient scale $[\sigma_w/(dU/dz)]^{-1}$ (after Hunt *et al.*, 1985).

atmospheric motions and temperature field. In section 3 we discuss whether the recent measurements of the SBL, particularly their variability, suggest any convenient parameterization of diffusion in the SBL based on gross properties of the SBL. In section 3.2 we look at some specific atmospheric diffusion data from elevated sources, the effects on diffusion of shear (both in speed and direction), and finally the problem of applying diffusion calculations to stack plumes.

2. Diffusion in idealized stably stratified turbulent shear flow

a. Kinematics of fluid elements

The main object of the study of the diffusion of a substance (S) through a turbulent flow is to relate the development of the concentration distribution $c(x_i)$ and fluxes of S, $\overline{cu}_i$ to the mean and turbulent velocity field $u_i(x_i)$ and to the transport processes. [It may be important in stably stratified flows to distinguish between the velocity and displacement of *fluid elements or particles* X $(=X, Y, Z)$ and those of *molecules* of (S)]. (See Fig. 2a.)

In this section we consider X and the distribution of c in a shear flow with mean velocity $[U(z), V(z)]$ and rms turbulence components σ_u, σ_v, σ_w, where the mean potential temperature gradient $d\bar{\theta}/dz$ is positive. The depth h of a stably stratified boundary layer is large compared with the length scale $L_x^{(w)}$ of the vertical turbulence (Fig. 1). Typically $L_x^{(w)} \sim \sigma_w/(dU/dz) \sim h\sigma_w/U(h) \ll h/10$ (Hunt *et al.*, 1985). Therefore the mean and turbulent flow field can be regarded as slowly varying functions of height, i.e., locally homogeneous. In most practical calculations one is interested in diffusion over a sufficiently large scale that the time $t(\geq h/U)$ for a plume or particles to travel from their source at x_s is comparable or large compared with $T_L(\sim L_x^{(w)}/\sigma_w)$, the Lagrangian timescale of the turbulence.

G. I. Taylor's theory gives the rate of growth of the mean square displacement of fluid elements as

$$\frac{d}{dt}\Sigma_z^2 = \frac{d}{dt}(\overline{Z^2}) = 2\left[\sigma_w^{(L)}(t)\int_0^t \sigma_w^{(L)}(t')R_w^{(L)}(t, t')dt'\right], \quad (1)$$

where $\sigma_w^{(L)}(t) = (\overline{w^{(L)2}(t)})^{1/2}$ and $w^{(L)}(t)$ is the fluctuating Lagrangian velocity of *fluid elements* which passed through the source at height z_s, and $R_w^{(L)} = \overline{w^{(L)}(t)w^{(L)}(t')}/\sigma_w^{(L)}(t)\sigma_w^{(L)}(t')$ is the Lagrangian autocorrelation coefficient. If $w^{(L)}(t)$ is a stationary random process, (1) reduces to

$$d\Sigma_z^2/dt = 2\sigma_w^{(L)2}\int_0^t R_w^{(L)}(t, t')dt', \quad (2)$$

and when $t \geq 2T_L$,

$$d\Sigma_z^2/dt \approx 2\sigma_w^{(L)2}T_L = 2K_z', \quad (3)$$

where

$$T_L = \int_0^\infty R_w^{(L)}(t - t')d(t - t'), \quad (4)$$

and K_z is the vertical diffusivity for the motion of fluid elements. The result (3) can be used if K_z varies slowly over the vertical distance through which particles diffuse in a time of order T_L, as is the case in neutral and stable conditions. If the turbulence is homogeneous over a scale that is *large* compared with Σ_z, $w^{(L)}$ can be regarded as a stationary process over a time that is *large* compared with T_L. Only then (2) can be integrated to give

$$\Sigma_z^2 \sim 2\sigma_w^{(L)2} \times \left[T_L t - \int_0^\infty (t - t')R_w^{(L)}(t - t')d(t - t')\right]. \quad (5)$$

The practical problems raised by this analysis are: i) what is T_L or K_z in any given situation; and ii) how can they be estimated from fixed-point measurements, either of the velocity field, or of the diffusion of another scalar such as heat and moisture; and iii) what happens to the plume near the ground?

b. *Dynamics and mixing processes for fluid elements*

In neutral stratification, physical arguments (including dimensional analysis) and experiments show that $T_L = bL_x^{(w)}/\sigma_w$ where b does not vary much more than a factor of 2 between different unidirectional turbulent shear flows. But in stable stratification with buoyancy frequency N, b is a function of the local dimensionless group $NL_x^{(w)}/\sigma_w$ and perhaps other parameters as well.

In a turbulent flow with neutral stability, fluid elements from a source move vertically over an ever-increasing distance as time proceeds, as shown by (3). However, in stably stratified flows an increasing vertical displacement of fluid elements requires a steady input of energy (assuming the density of the fluid element remains unchanged) (Fig. 2). Given i) a finite total amount of energy in the turbulence and that the turbulent velocity is a stationary random process; ii) an average kinetic energy of fluid elements of the order of $\frac{1}{2}\rho\sigma_w^2$; and iii) a roughly equal partition of energy between kinetic and potential energy (demonstrable by measurement and calculation), it follows that fluid elements can only be displaced through a vertical distance Z of order σ_w/N where N is the Brunt-Väisälä frequency. So

$$d\overline{Z^2}/dt = 0 \quad \text{and} \quad \overline{Z^2} = \Sigma_z^2 \sim \sigma_w^2/N^2.$$

However, as an element rises into warmer air, the combination of small-scale turbulence and molecular diffusion result in an increase of its temperature. Only by changes in their temperature can fluid elements diffuse above and below the limiting depth of order σ_w/N. But such molecular exchange is also likely to *dilute* the concentration of contaminant marking the fluid, as we see later. Note that in a turbulent flow the magnitude of the molecular exchange of temperature between fluid elements does not depend directly on the value of the molecular diffusivity because turbulence generates motions at small scales, small enough that the molecular diffusion locally balances advection by the turbulence, as in other high Reynolds number turbulent processes.

One process that can cause such mixing between displaced fluid elements and their environment is the overturning of internal waves (Fig. 3a), for which the time scale is proportional to N^{-1}, but is large compared with N^{-1} because such mixing events are not frequent. This is one physical mechanism to explain why the time for finite elements to change their density (or temperature) is larger by a factor of γ^{-1} than the time N^{-1} of a typical up-and-down oscillation, where $\gamma \ll 1$. Such a hypothesis clearly implies that in such a motion the change in the potential temperature $\Delta\theta$ of

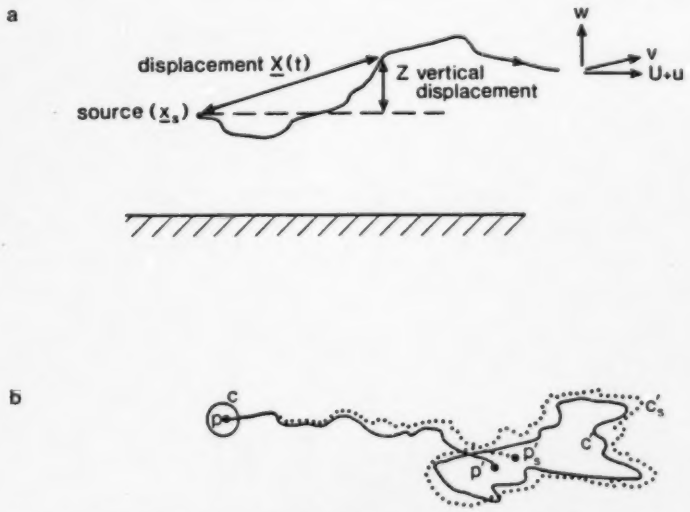

FIG. 2. (a) Notation for source location, displacement and velocity of a fluid element leaving the source at $t = 0$. (b) Sketch showing the difference between the trajectory of marked fluid elements and the trajectory of molecules of S. Curve C is the envelope of marked fluid elements and molecules of S at $t = 0$. The points C', C'_s are the envelopes of the fluid elements and the molecules (in a mean square sense) at a particular time t later. A typical fluid element moves from P to P' and a molecule from P to P'_s.

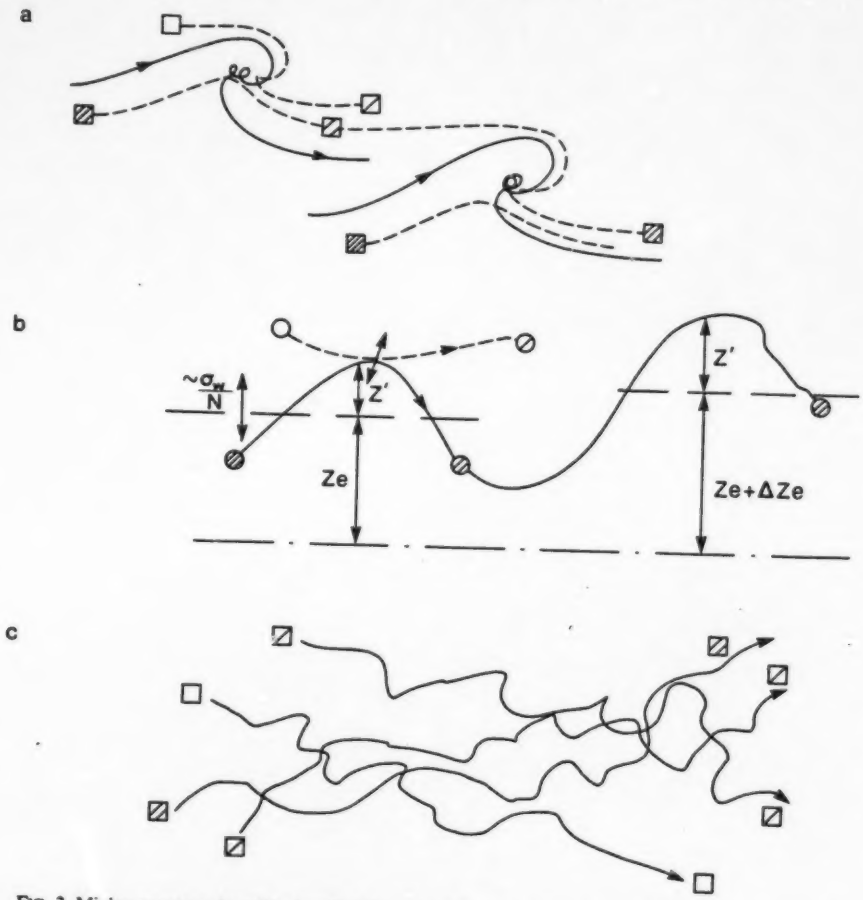

FIG. 3. Mixing processes in stably stratified flows. (a) Effect of breaking internal waves on the density of fluid elements, showing how a density (or temperature) flux is developed. (b) Idealization of fluid elements oscillating and changing their density after mixing with the other fluid elements; the equilibrium level of a fluid elements changes from Z_e to ΔZ_e (after Pearson *et al.* 1983). (c) Mechanism of a temperature or concentration flux in neutral or weakly stratified flows by advection of fluid elements.

a particle as it moves up and down over a distance $\Sigma_z(\sim\sigma_w/N)$ can be estimated as $\Delta\theta \sim \gamma\Sigma_z\partial\bar{\theta}/\partial z \sim \gamma(\sigma_w/N)\partial\bar{\theta}/\partial z$. This increase in $\Delta\theta$ is correlated with a downward or negative velocity, so the temperature flux $F_\theta = \overline{w\theta}$ can be estimated as

$$F_\theta \sim -\Delta\theta\sigma_w \sim (\sigma_w^2/N)(\partial\bar{\theta}/\partial z); \tag{6a}$$

therefore,

$$K_\theta \sim \gamma\sigma_w^2/N. \tag{6b}$$

When through mixing the potential temperature of a fluid element changes by $\Delta\theta$, the equilibrium level of marked particle changes, from Z_e to $Z_e + \Delta Z_e$ during a typical up-and-down oscillation, where $\Delta Z_e = \Delta\theta/\partial\bar{\theta}/\partial z \sim \gamma\sigma_w/N$ (see Fig. 3b). Since the particle now moves randomly (through $\pm\sigma_w/N$) relative to its *new* equilibrium height, we write

$$Z = Z_e + Z'$$

where Z' is approximately independent of Z_e and is the displacement generated without mixing. So $d\overline{Z'^2}/dt \approx 0$ and therefore

$$\frac{d\Sigma_z^2}{dt} = \frac{d\overline{Z^2}}{dt} = \frac{d\overline{Z_e^2}}{dt} + \frac{d\overline{Z'^2}}{dt} \approx d\overline{z_e^2}/dt. \tag{7}$$

Note that this rate of growth of Σ_z^2 is *not* equal to the diffusivity K_z, as in neutrally stratified turbulence. This is explained below.

Mathematical models of these diffusion and mixing processes have been developed by Csanady (1969) and Pearson *et al.* (1983). Pearson *et al.* show that:

i) Near the source, when $t \ll N^{-1}$,

$$\Sigma_z^2 = \sigma_w^2 t^2, \tag{8a}$$

(For homogeneous turbulence $\sigma_w = \sigma_w^{(L)}$.)

ii) Far downwind when $t \gg N^{-1}$,

$$\Sigma_z^2 = (\sigma_w/N)^2[\zeta_\infty^2 + 2tT_LN^2] \quad (8b)$$

where ζ_∞^2 is a parameter of order unity, which is determined by the form of the Lagrangian spectrum of the fluctuating vertical pressure gradient.

If the mixing of temperature or density between fluid elements takes place over a timescale $(\gamma N)^{-1}$, the model and the estimate (7) show that $T_L \sim \gamma^2 N^{-1}$. So if $\gamma \approx 0.1$, the plume effectively levels out with thickness $\Sigma_z \sim \sigma_w/N$, but if $\gamma \sim 0.3$, the plume essentially grows parabolically after travelling for a time of order $2N^{-1}$. (In the limit of zero small-scale mixing $\gamma = 0$, $T_L = 0$ and in (4) we see that if Σ_z^2 is finite when $\tau \to \infty$, then the Lagrangian autocorrelation must be negative for large time, i.e., $\int_0^\infty \tau R_w^{(L)}(\tau)d\tau < 0$. (Pasquill 1974, p. 201).

iii) In stable or neutral conditions, temperature fluctuations (whose rms values are denoted by σ_θ) are produced by fluid elements with different temperatures passing the measuring point M (Fig. 4). In neutral conditions where fluid elements travel large distances, σ_θ is limited by the mixing between fluid elements. But in stable conditions, where the fluid elements are displaced over small distances, σ_θ is *not* a sensitive function of mixing, and so must be simply proportional to Σ_z or σ_w/N, i.e.,

$$\sigma_\theta = \zeta_\theta(\sigma_w/N)d\bar{\theta}/dz, \quad (9a)$$

where ζ_θ is a parameter of order unity. Inspection of Fig. 4 shows that σ_θ (or ζ_θ) is only affected by fluid elements with temperatures different from the mean temperature at M, while contributions to Σ_z (or ζ_∞) come from *all* fluid elements passing through M. The model of Pearson *et al.* (1983) predicts

$$\zeta_\theta = \frac{1}{\sqrt{2}}\zeta_\infty. \quad (9b)$$

The result (9a) is consistent with the second-order model of Brost & Wyngaard (1978) and observations (e.g., Hunt *et al.*, 1985; Nieuwstadt 1984).

iv) In any locally homogeneous flow the temperature flux, F_θ, or flux of the substance S, F_S, measured at a point is given by the correlation of the local vertical velocity and the local fluctuation

$$F_\theta = \overline{\theta' u_3}; \quad (10a)$$

but in *Lagrangian* terms, F_θ has two components: the random motion of fluid elements with different temperatures and transfer of heat between fluid elements by small-scale turbulence and molecular diffusion. Thus, in a homogeneous uniformly stratified flow

$$F_\theta = -\frac{1}{2}\frac{d\Sigma_z^2}{dt}\frac{d\bar{\theta}}{dz} + \overline{\Delta\theta(t)w^{(L)}(t)}, \quad (10b)$$

where $\Delta\theta$ is the change in the temperature of a fluid element (not the fluctuating temperature measured at a point θ') and $w^{(L)}(t)$ is its velocity. In neutral flows the latter effect is much smaller than the former; then (10) can be written

$$F_\theta = -K_\theta d\bar{\theta}/dz$$

where the diffusivity $K_\theta = K_z = \frac{1}{2}d\Sigma_z^2/dt$. In this case the flux of heat is transported by the unlimited displacements of fluid elements across the flow, indicated by Fig. 3c; K_z can be calculated in terms of the turbulent velocity from (3).

But in stably stratified flows, the mean square vertical displacement of the fluid elements grows very slowly, so that from (7)

$$\frac{d\Sigma_z^2}{dt}\left(-\frac{d\bar{\theta}}{dz}\right) \sim \frac{\gamma^2\sigma_w^2}{N}\left(-\frac{d\bar{\theta}}{dz}\right).$$

But the contribution from the mixing between fluid elements is

$$\overline{\Delta\theta(t)w^{(L)}(t)} \sim \frac{\gamma\sigma_w^2}{N}\left(-\frac{d\bar{\theta}}{dz}\right).$$

Thus, if $\gamma \ll 1$, the second term is $O(\gamma^{-1})$, larger than the first term, and most of the temperature flux is caused by the fluid elements exchanging temperature during their oscillations making the second term in (10b) the larger contributor of F_θ and thence to the diffusivity K_θ. In other words in stable conditions the exchange mechanism in Fig. 3b is more effective than the usual transport mechanism depicted in Fig. 3c. A further consequence is that from measurements of F_θ in the atmosphere, one cannot in general derive the rate of increase $d\Sigma_z^2/dt$ of the displacements of fluid elements, because $-\frac{1}{2}(d\Sigma_z^2/dt)(d\bar{\theta}/dz)$ is much less than F_θ.

Many models assume or predict that in strongly stable conditions the local diffusivity of temperature is determined by the local values of σ_w and N, i.e.,

$$K_\theta = \gamma_\theta\sigma_w^2/N \quad (11)$$

where

$$F_\theta = -K_\theta d\bar{\theta}/dz$$

and γ_θ is a "constant" of the order unity. The model of Pearson *et al.* (1983) shows that γ_θ depends on the form of the *spectra* as well as on the parameter γ determining the mixing between fluid elements.

But the model also shows that when $\gamma \ll 1$, the de-

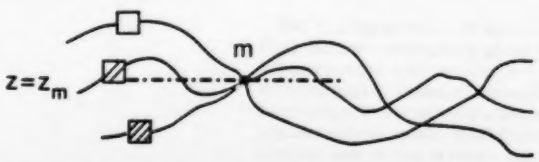

FIG. 4. Fluid elements passing through the measuring position M. Upper box: denser; Middle box: less dense; Lower box: approximately same density as the mean density for temperature at $z = Z_M$.

if $F_s = -K_s dC/dz$. In stable conditions if the mixing between fluid elements is slow (i.e., $\gamma \ll 1$), the source substance S is advected by the fluid elements passing through the source, so that the displacements of the fluid elements determine the concentration distribution, i.e., for $t \ll (\gamma N)^{-1}$

$$\sigma_z^2 = \Sigma_z^2. \tag{16a}$$

(This corresponds to the time range for diffusion measurements by Britter *et al.* (1983) in laboratory grid turbulence experiments.) Since the rate of increase of Σ_z^2 is very small when $t \gtrsim N^{-1}$, it follows that when $(\gamma N)^{-1} \gg t \gtrsim N^{-1}$,

$$\frac{d\sigma_z^2}{dt} = 2K_s \sim \gamma^2 \sigma_w^2 / N. \tag{16b}$$

However, over a long enough period of travel from the source such that $t \gg (\gamma N)^{-1}$, sufficient mixing can occur between the marked and other fluid elements that the flux of S becomes *independent* of the displacement Z of those original marked fluid elements and the growth of the plume can become greater than that of the displacement of fluid elements. Figure 5a depicts the process of the marked fluid elements exchanging temperature and S with the surroundings and thus increasing their vertical displacements, the diffusion of S and the decoupling of the diffusion S from the motion of the original marked fluid elements. Then the flux of S must be determined by the local motions and mixing of fluid elements within a limited vertical distance $(\pm\sigma_w/N)$, and must be proportional to the local values of $\partial C/\partial z$. Since for high Reynolds number turbulence flows the mixing only depends on the turbulence properties, the diffusivities of heat and of the substance S are the same, i.e., when $t \gg (\gamma N)^{-1}$

$$K_\theta = K_s \gg \tfrac{1}{2} d\Sigma_z^2/dt, \tag{17a}$$

whereas in neutral flows, when $t \gg T_L$,

$$K_\theta = K_s = \tfrac{1}{2} d\Sigma_z^2/dt. \tag{17b}$$

Thus in stably stratified flow it is not in general correct to express the results for $d\sigma_z^2/dt$ in terms of $d\Sigma_z^2/dt$ or in terms of T_L.

Many investigators have assumed the validity of (17b) in stably stratified flows, but have in fact only used the assumption that $K_\theta = K_s$, (e.g., Chaudhry and Meroney, 1973; Doran *et al.*, 1978; Venkatram *et al.*, 1984). They have found that this equivalence leads to a useful way of estimating $C(x)$ downwind of a source, given boundary layer measurements of K_θ. It is interesting to note that this prediction is implicit in diffusivity models (e.g., $k - \epsilon$ models) (Rodi, 1985). Distinctions between diffusivities of heat and released substances are made in some second order closure models, but even then these models do not describe the initial stages of plume growth (i.e., $t \ll (\gamma N)^{-1}$) (Lumley and Mansfield, 1984).

The arguments leading to (17b) are a straightforward but new deduction from the ideas of Pearson *et al.* (1983). They did not consider the growth of plumes when $t \gtrsim (\gamma N)^{-1}$ and therefore emphasized the differences between K_θ and K_z near the source. There is still no laboratory or field data that we know of in stably stratified flows far enough downwind of a source to test these ideas in detail (i.e., when $t \gtrsim 10\ \mathrm{N}^{-1}$, or typically 1 km.) Only Lagrangian measurements of fluid elements marked at the sources and concentration measurements could detect differences between K_z and K_s.

3. Diffusion in the stable boundary layer

a. Is the structure of the SBL too variable to rely on?

There are four main characteristic features of the stably stratified boundary layer:

i) Its turbulent intensity is small ($\sigma_w/U \ll 1$), which means that the shear has to be large.

ii) The scales of turbulence are small compared with the boundary-layer height ($L_x^{(w)}/h \ll 1$).

iii) Any disturbances interact with the temperature gradients in it and above it to produce internal gravity waves whose amplitudes vary with height according to the temperature distribution, which is usually *not* on the same scale as the boundary-layer thickness h.

iv) Any small changes in local topography or local weather conditions affect the boundary layer for a long way downwind. [Even slopes of 10^{-3} may have significant effects on the wind profiles in stable conditions (Garratt, 1981; Brost and Wyngaard, 1978)].

In the recent major investigation of the SBL by Caughey *et al.* (1979), it was found that only by removing the low frequency wave part of the spectrum of w, θ, $w\theta$, etc., was it possible to find any repeatable structure of the turbulence (see p. 1043 of their paper). The part of the spectra that they considered probably provides most of the momentum transfer, but for mass and heat transfer, the *low frequency* part of the spectrum is equally important. It is a striking feature of the B.A.O. data that *wave motion* appears to be connected with this low frequency heat transfer, which contradicts some simple ideas of wave motion.

If, even on the plains of Minnesota, this part of the spectrum has no repeatable structure, how much more true is that elsewhere, as the measurements by Hunt *et al.* (1983) at B.A.O., and Nieuwstadt (1984) at Cabauw indicate? For example, variance calculated from the entire, unfiltered spectra of w, θ and $w\theta$ often do not decrease significantly with height, although filtered $w\theta$ usually does.

The SBL is quite different from convective boundary layers (CBL) where the strong local turbulence, larger turbulence scale, and lack of wave motion ensure that the turbulence is determined by local parameters and its structure can therefore be universal. Despite the success of Nieuwstadt (1984) in developing a calcula-

tion for the stable boundary-layer structure with no lateral variations, I am doubtful about the value of expressing diffusion parameters, for example, σ_z, σ_y or K_S, etc. as universal functions of h/L_{MO}, (Z_s/h, x/h etc.) where L_{MO} is the local Monin–Obukhov length. However if local measurements or estimates of σ_w, N, σ_θ, $w\theta$ are available, then statistical diffusion calculations (based in part on the ideas outlined in section 2) may be appropriate. Thus in the SBL it is particularly important to have models for the diffusion that are consistent with a wide range of possible mean and turbulent flows.

b. *Analysis of a range of plume phenomenon in the SBL*

Recent measurements at B.A.O. and Cabauw have provided information about the temperature fluctuations and heat flux throughout the whole depth of the nighttime SBL (Hunt, Kaimal and Gaynor 1985; Nieuwstadt 1984). They show similar values for the ratios $\zeta_\theta[=\sigma_\theta/(\sigma_w/N\cdot d\bar{\theta}/dz)] \approx (0.75 \pm 0.25)$ and $\gamma_\theta = K_\theta/(\sigma_w^2/N) \approx (0.2 \pm 0.1)$. These measurements can be used with the statistical (or other) models of turbulent diffusion to infer the *range* of values of σ_z which might be expected in these conditions. It is important to emphasize the significance of the variation of parameter γ_θ, because physical arguments and the model of Pearson *et al.* (1983) indicate that when $\gamma_\theta \ll 1$, $(\gamma_\theta N)^{-1}$ determines the rate of mixing between fluid elements (i.e., $\gamma_\theta \approx \gamma$) and that the rate of increase of Σ_z^2 when $t \gtrsim N^{-1}$ varies as γ_θ^2. So when $\gamma_\theta \approx 0.1$, the plume depth grows to a depth of about σ_w/N and is then effectively constant for a distance from the source at least of order $U/(\gamma N)$, i.e.,

$$\Sigma_z^2 \sim \zeta_z^2\sigma_w^2/N^2, \quad \text{where} \quad 1.0 \leqslant \zeta_z \leqslant 1.4, \quad (18a)$$

(which may be 10 km or more).

The model suggests that a significant difference develops between Σ_z^2 and σ_z^2 beyond these distances. However, when $\gamma_\theta \approx 0.3$, the plume depth and Σ_z effectively increase as the square root of the distance downwind, within a distance of order $(3U/N)$ of the source. At this distance, the rate of growth of σ_z^2 becomes approximately proportional to the thermal diffusivity K_θ, so that

$$d\sigma_z^2/dt \approx 2K_\theta \approx 2\gamma_\theta\sigma_w^2/N, \quad (18b)$$

as proposed by Venkatram *et al.* (1984); Draxler (1976) and others.

Some authors have assumed that (18b) is generally valid in stable conditions, e.g., Venkatram *et al.* (1984). Experimental evidence (including that of Venkatram *et al.*) indicates that on some occasions (16a) is a better description and on others (16b). (See Hunt 1982 and Pearson *et al.* 1983 for details.) A proper test of these ideas requires *simultaneous* measurements of heat flux and temperature fluctuations (i.e., K_θ and σ_θ) and of the plume depth σ_z. One hopes that current field studies

FIG. 6. The development of the cross section of a plume in a turbulent stratified environment showing how it can grow from its initial cross section or slump depending on the relative strengths of the turbulence and the stratification.

(e.g., those planned in the Netherlands and the United States) will rectify this omission.

Near the surface, even in stable conditions, there is less scatter in the structure of vertical turbulence and the thermal and temperature fluctuations. It has been shown by Nieuwstadt and van Ulden (1978) and others that eddy diffusivity calculations are quite satisfactory for sources near the ground. The probable reason is that shear ensures a high value of γ_θ ($\sim$0.3) so that mixing is rapid and then $K_\theta \approx K_z$.

An important feature of the SBL is the presence of large-shear in the senses of relative rapid changes of speed and direction with height, e.g., by 40° in 100 m. If the vertical diffusion can be modeled by a diffusivity K_z, then, as Doran *et al.* (1978) have shown, the effect of shear dU/dz is to lower the height z_m of the maximum concentration by about $(\sigma_z^2/2)(dU/dz)/U$, an effect neglected in Gaussian plume modeling. The effect of lateral shear dV/dz increases the rate of growth, $d\sigma_y^2/dt$, of the square of the horizontal width of the plume in proportion to

$$K_z l^2(\partial V/\partial z)^2, \quad (19a)$$

which of course continues to increase downwind and may exceed the contribution by the lateral component of turbulence σ_v. (Slight topographic features or thermal gradients can strongly affect dV/dz). If σ_z levels out (as is often the case over water), then

$$\frac{d\sigma_y^2}{dt} \sim \sigma_z^2(\partial V/\partial z)^2. \quad (19b)$$

Most diffusion calculations are applied to plumes released from stacks in which the temperature and concentration of S are initially well mixed over a scale σ_{z_0}. In the presence of a weak atmospheric turbulence and a stable temperature gradient, σ_{z_0} may be greater than (σ_w/N). In that case the buoyancy forces within the plume tend to make it "slump" outwards horizontally and reduce its vertical scale. These can overcome the tendency of the vertical component of atmospheric turbulence to thicken it as shown in Fig. 6. The turbulence-induced growth of the plume also depends on the rate of mixing between fluid elements in the flow. Both kinds of plume behavior shown in Fig. 5 are observed.

Acknowledgments. I should like to thank the organizers and participants of the AMS Workshop for an

excellent meeting and for their comments on this paper which, of course, also reflects many conversations with colleagues at Cambridge and at Boulder.

APPENDIX

Nomenclature

Symbols

b	constant
c	concentration
C	mean concentration
F_θ, F_S	flux of temperature and substance S
h	boundary-layer depth
k	kinetic energy
K	eddy diffusion
$L_x^{(w)}$	integral scale of vertical velocity
N	buoyancy frequency
R_w^L	autocorrelation
S	substance
T_L	Lagrangian time scale
t	time
u	velocity
U	mean velocity
$w^{(L)}$	Lagrangian velocity
x, x_i $(=x, y, z)$	position
X, $(=X, Y, Z)$	displacement
λ	mixing coefficient
ϵ_θ	dissipation rate
ζ	dimensionless parameter
κ	thermal diffusivity
θ	temperature
ρ	density
Σ_z	rms displacement of fluid elements
$\sigma_u\sigma_v\sigma_w$	rms velocity components
σ_y	horizontal scale of diffusion of plume
σ_z	vertical scale of diffusion of plume

Prefix

Δ	difference

Sub-superscript

e	equilibrium
L	Lagrangian
θ	temperature
z	vertical for fluid elements
s	substance
∞	asymptotic value

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